

LMU 20/03
February 2, 2008

Does the QCD Scale vary in time ?

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Abstract

Last year I talked at this meeting about a possible time dependence of the QCD coupling constant α_s . This year I shall look into this problem once more, without fully repeating the arguments given last year. Astrophysical indications that the fine structure constant has undergone a small time variation during the cosmological evolution are discussed within the framework of the standard model of the electroweak and strong interactions and of grand unification. A variation of the electromagnetic coupling constant could either be generated by a corresponding time variation of the unified coupling constant or by a time variation of the unification scale, or by both. The various possibilities, differing substantially in their implications for the variation of low energy physics parameters like the nuclear mass scale, are discussed. The case in which the variation is caused by a time variation of the unification scale is of special interest. It is supported in addition by recent hints towards a time change of the proton-electron mass ratio.

The Standard Model of the electroweak and strong interactions has about 18 parameters, which have to be adjusted in accordance with experimental observations. These include the three coupling strengths g_1 , g_2 , g_3 , the scale of the electroweak symmetry breaking, given by the universal Fermi constant, the 9 Yukawa couplings of the six quarks and the three charged leptons, and the four electroweak mixing parameters. One parameter, the mass of the hypothetical scalar boson, is still undetermined. For the physics of stable matter, i.e. atomic physics, solid state physics and a large part of nuclear physics, only six constants are of importance: the mass of the electron, the masses of the u and d -quarks setting the scale of the breaking of isotopic spin, and the strong interaction coupling constant α_s . The latter, often parametrized by the QCD scale parameter Λ , sets also the scale for the nucleon mass.

The mass of the strange quark should also be included since the mass term of the s -quarks is expected to contribute to the nucleon mass, although the exact amount of strangeness contribution to the nucleon mass is still being discussed - it can range from several tenth of MeV till more than 100 MeV. As far as macro-physical aspects are concerned, Newton's constant must be added, which sets the scale for the Planck units of energy, space and time.

Since within the Standard Model the number of free parameters cannot be reduced, and thus far theoretical speculations about theories beyond the model have not led to a well-defined framework, in view of lack of guidance by experiment, one may consider the possibility that these parameters are time and possibly also space variant on a cosmological scale.

Speculations about a time-change of coupling constants have a long history, starting with early speculations about a cosmological time change of Newton's constant G [1–4]. Since in particular the masses of the fermions as well as the electroweak mass scale are related to the vacuum expectation values of a scalar field, time changes of these parameters are conceivable. In some theories beyond the Standard Model also the gauge coupling constants are related to expectation values of scalar fields which could be time dependent [5].

Recent observations in astrophysics concerning the atomic fine-structure of elements in distant objects suggest a time change of the fine structure constant [6]. The data suggest that α was lower in the past, at a redshift of

$z \approx 0.5 \dots 3.5$. More recent observations give:

$$\Delta\alpha/\alpha = (-0.72 \pm 0.18) \times 10^{-5}. \quad (1)$$

If α is indeed time dependent, the other two gauge coupling constants of the Standard Model are also expected to depend on time, as pointed out recently [7] (see also [8,9]), if the Standard Model is embedded into a grand unified theory. Moreover the idea of a grand unification of the coupling constants leads to a relation between the time variation of the electromagnetic coupling constant and the QCD scale parameter Λ , implying a physical time variation of the nucleon mass, when measured in units given by an energy scale independent of QCD, like the electron mass or the Planck mass. The main assumption is that the physics responsible for a cosmic time evolution of the coupling constants takes place at energies above the unification scale. This allows to use the usual relations from grand unified theories to evolve the unified coupling constant down to low energy. Of particular interest is the relatively large time change of the proton mass in comparison to the time change of α , which will be discussed below.

Considering the six basic parameters mentioned above plus Newton's constant G , one can in general consider seven relative time changes: \dot{G}/G , $\dot{\alpha}/\alpha$, $\dot{\Lambda}/\Lambda$, \dot{m}_e/m_e , \dot{m}_u/m_u , \dot{m}_d/m_d and \dot{m}_s/m_s . Thus in principle seven different functions of time do enter the discussion. However not all of them could be measured. Only dimensionless ratios, e.g. the ratio Λ/m_e or the fine-structure constant could be considered as candidates for a time variation.

The time derivative of the ratio Λ/m_e describes a possible time change of the atomic scale in comparison to the nuclear scale. In the absence of quark masses there is only one mass scale in QCD, unlike in atomic physics, where the two parameters α and m_e enter. The parameter α is directly measurable by comparing the energy differences describing the atomic fine structure (of order $m_e c^2 \alpha^4$) to the Rydberg energy $hcR_\infty = m_e c^2 \alpha^2 / 2 \approx 13.606$ eV.

Both astrophysics experiments as well as high precision experiments in atomic physics in the laboratory could in the future give indications about a time variation of three dimensionless quantities: α , M_p/m_e and $(M_n - M_p)/m_e$. The time variation of α reported in [6] implies, assuming a simple linear extrapolation, a relative rate of change per year of about $1.0 \times 10^{-15}/\text{yr}$. This poses a problem with respect to the limit given by an analysis of the remains of

the naturally occurring nuclear reactor at Oklo in Gabon (Africa), which was active close to 2 billion years ago. One finds a limit of $\dot{\alpha}/\alpha = (-0.2 \pm 0.8) \times 10^{-17}/\text{yr}$. This limit was derived in [11] under the assumption that other parameters, especially those related to the nuclear physics, did not change during the last 2 billion years. It was recently pointed out [7, 10], that this limit must be reconsidered if a time change of nuclear physics parameters is taken into account. In particular it could be that the effects of a time change of α are compensated by a time change of the nuclear scale parameter. For this reason we study in this paper several scenarios for time changes of the QCD scale, depending on different assumptions about the primary origin of the time variation.

Without a specific theoretical framework for the physics beyond the Standard Model the relative time changes of the three dimensionless numbers mentioned above are unrelated. We shall incorporate the idea of grand unification and assume for simplicity the simplest model of this kind, consistent with present observations, the minimal extension of the supersymmetric version of the Standard Model (MSSM), based on the gauge group $SU(5)$. In this model the three coupling constants of the Standard Model converge at high energies at the scale Λ_G . In particular the QCD scale Λ and the fine structure constant α are related to each other. In the model there are besides the electron mass and the quark masses three further scales entering, the scale for the breaking of the electroweak symmetry Λ_w , the scale of the onset of supersymmetry Λ_s and the scale Λ_G where the grand unification sets in.

Assuming $\alpha_u = \alpha_u(t)$ and $\Lambda_G = \Lambda_G(t)$, one finds:

$$\frac{1}{\alpha_i} \frac{\dot{\alpha}_i}{\alpha_i} = \left[\frac{1}{\alpha_u} \frac{\dot{\alpha}_u}{\alpha_u} - \frac{b_i^S}{2\pi} \frac{\dot{\Lambda}_G}{\Lambda_G} \right] \quad (2)$$

which leads to

$$\frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} = \frac{8}{3} \frac{1}{\alpha_s} \frac{\dot{\alpha}_s}{\alpha_s} - \frac{1}{2\pi} \left(b_2^S + \frac{5}{3} b_1^S - \frac{8}{3} b_3^S \right) \frac{\dot{\Lambda}_G}{\Lambda_G}. \quad (3)$$

One may consider the following scenarios:

- 1) Λ_G invariant, $\alpha_u = \alpha_u(t)$. This is the case considered in [7] (see also [8]), and one finds

$$\frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} = \frac{8}{3} \frac{1}{\alpha_s} \frac{\dot{\alpha}_s}{\alpha_s} \quad (4)$$

and

$$\frac{\dot{\Lambda}}{\Lambda} = -\frac{3}{8} \frac{2\pi}{b_3^{SM}} \frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha}. \quad (5)$$

- 2) α_u invariant, $\Lambda_G = \Lambda_G(t)$. One finds

$$\frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} = -\frac{1}{2\pi} \left(b_2^S + \frac{5}{3} b_1^S \right) \frac{\dot{\Lambda}_G}{\Lambda_G}, \quad (6)$$

with

$$\Lambda_G = \Lambda_S \left[\frac{\Lambda}{\Lambda_S} \exp \left(-\frac{2\pi}{b_3^{SM}} \frac{1}{\alpha_u} \right) \right]^{\left(\frac{b_3^{SM}}{b_3^S} \right)} \quad (7)$$

which follows from the extraction of the Landau pole. One obtains

$$\frac{\dot{\Lambda}}{\Lambda} = \frac{b_3^S}{b_3^{SM}} \left[\frac{-2\pi}{b_2^S + \frac{5}{3} b_1^S} \right] \frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} \approx -30.8 \frac{\dot{\alpha}}{\alpha} \quad (8)$$

- 3) $\alpha_u = \alpha_u(t)$ and $\Lambda_G = \Lambda_G(t)$. One has

$$\begin{aligned} \frac{\dot{\Lambda}}{\Lambda} &= -\frac{2\pi}{b_3^{SM}} \frac{1}{\alpha_u} \frac{\dot{\alpha}_u}{\alpha_u} + \frac{b_3^S}{b_3^{SM}} \frac{\dot{\Lambda}_G}{\Lambda_G} \\ &= -\frac{3}{8} \frac{2\pi}{b_3^{SM}} \frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} - \frac{3}{8} \frac{1}{b_3^{SM}} \left(b_2^S + \frac{5}{3} b_1^S - \frac{8}{3} b_3^S \right) \frac{\dot{\Lambda}_G}{\Lambda_G} \\ &= 46 \frac{\dot{\alpha}}{\alpha} + 1.07 \frac{\dot{\Lambda}_G}{\Lambda_G} \end{aligned} \quad (9)$$

where theoretical uncertainties in the factor $R = (\dot{\Lambda}/\Lambda)/(\dot{\alpha}/\alpha) = 46$ have been discussed in [7]. The actual value of this factor is sensitive to the inclusion of the quark masses and the associated thresholds, just like in the determination of Λ . Furthermore higher order terms in the QCD evolution of α_s will play a role. In ref. [7] it was estimated: $R = 38 \pm 6$.

The case in which the time variation of α is not related to a time variation of the unified coupling constant, but rather to a time variation of the unification scale, is of particular interest. Unified theories, in which the Standard Model arises as a low energy approximation, might well provide a numerical value for the unified coupling constant, but allow for a smooth time variation of the unification scale, related in specific models to vacuum expectation values of scalar

fields. Since the universe expands, one might expect a decrease of the unification scale due to a dilution of the scalar field. A lowering of Λ_G implies according to (6)

$$\frac{\dot{\alpha}}{\alpha} = -\frac{1}{2\pi}\alpha \left(b_2^S + \frac{5}{3}b_1^S \right) \frac{\dot{\Lambda}_G}{\Lambda_G} = -0.014 \frac{\dot{\Lambda}_G}{\Lambda_G}. \quad (10)$$

If $\dot{\Lambda}_G/\Lambda_G$ is negative, $\dot{\alpha}/\alpha$ increases in time, consistent with the experimental observation. Taking $\Delta\alpha/\alpha = -0.72 \times 10^{-5}$, we would conclude $\Delta\Lambda_G/\Lambda_G = 5.1 \times 10^{-4}$, i.e. the scale of grand unification about 8 billion years ago was about 8.3×10^{12} GeV higher than today. If the rate of change is extrapolated linearly, Λ_G is decreasing at a rate $\frac{\dot{\Lambda}_G}{\Lambda_G} = -7 \times 10^{-14}/\text{yr}$.

According to (8) the relative changes of Λ and α are opposite in sign. While α is increasing with a rate of $1.0 \times 10^{-15}/\text{yr}$, Λ and the nucleon mass is decreasing, Λ and the nucleon mass are decreasing, e.g. with a rate of $1.9 \times 10^{-14}/\text{yr}$. The magnetic moments of the proton μ_p as well of nuclei would increase according to

$$\frac{\dot{\mu}_p}{\mu_p} = 30.8 \frac{\dot{\alpha}}{\alpha} \approx 3.1 \times 10^{-14}/\text{yr}. \quad (11)$$

The effect can be seen by monitoring the ratio $\mu = M_p/m_e$. Measuring the vibrational lines of H_2 , a small effect was seen recently [12]. The data allow two different interpretations:

- a) $\Delta\mu/\mu = (5.7 \pm 3.8) \times 10^{-5}$
- b) $\Delta\mu/\mu = (12.5 \pm 4.5) \times 10^{-5}$.

The interpretation b) agrees with the expectation based on (8):

$$\frac{\Delta\mu}{\mu} = 22 \times 10^{-5}. \quad (12)$$

It is interesting that the data suggest that μ is indeed decreasing, while α seems to increase. If confirmed, this would be a strong indication that the time variation of α at low energies is caused by a time variation of the unification scale.

The time variation of the ratio M_p/m_e and α discussed here are such that they could be discovered by precise measurements in quantum optics. The wave length of the light emitted in hyperfine transitions, e.g. the ones used in the

cesium clocks being proportional to $\alpha^4 m_e/\Lambda$ will vary in time like

$$\frac{\dot{\lambda}_{hf}}{\lambda_{hf}} = 4 \frac{\dot{\alpha}}{\alpha} - \frac{\dot{\Lambda}}{\Lambda} \approx 3.5 \times 10^{-14}/\text{yr} \quad (13)$$

taking $\dot{\alpha}/\alpha \approx 1.0 \times 10^{-15}/\text{yr}$ [6]. The wavelength of the light emitted in atomic transitions varies like α^{-2} :

$$\frac{\dot{\lambda}_{at}}{\lambda_{at}} = -2 \frac{\dot{\alpha}}{\alpha}. \quad (14)$$

One has $\dot{\lambda}_{at}/\lambda_{at} \approx -2.0 \times 10^{-15}/\text{yr}$. A comparison gives:

$$\frac{\dot{\lambda}_{hf}/\lambda_{hf}}{\dot{\lambda}_{at}/\lambda_{at}} = -\frac{4\dot{\alpha}/\alpha - \dot{\Lambda}/\Lambda}{2\dot{\alpha}/\alpha} \approx -17.4. \quad (15)$$

At present the time unit second is defined as the duration of 6.192.631.770 cycles of microwave light emitted or absorbed by the hyperfine transition of cesium-133 atoms. If Λ indeed changes, as described in (8), it would imply that the time flow measured by the cesium clocks does not fully correspond with the time flow defined by atomic transitions.

It remains to be seen whether the effects discussed in this paper can soon be observed in astrophysics or in quantum optics. A determination of the double ratio $(\dot{\Lambda}/\Lambda)/(\dot{\alpha}/\alpha) = R$ would be of crucial importance, both in sign and in magnitude. If one finds the ratio to be about -20 , it would be considered as a strong indication of a unification of the strong and electroweak interactions based on a supersymmetric extension of the Standard Model. In any case the numerical value of R would be of high interest towards a better theoretical understanding of time variation and unification.

Recently a high precision experiment was done at the MPQ in Munich. The preliminary results is consistent with no change of the frequencies - one measures $2.8(5.7) \cdot 10^{-15}$ per year [13].

According to eq. (15) the effect should be about ten times larger. Although this result is still preliminary, one is supposed to think what might be the reason for the small effect.

One possibility is, of course, that the astrophysical measurements of the change of α are not correct. Another interesting possibility, however, needs to be studied. It might be that both α_{uu} and Λ_G change such that the result of λ_{hf} is exxentially zero - both effects cancel each other in leading order. Nevertheless on the level of 10^{-15} an effect should be seen. More refined experiments are needed to search for a time dependence of Λ .

Acknowledgements

We should like to thank G. Boerner, T. Haensch, M. Jacob, P. Minkowski, H. Walther and A. Wolfe for useful discussions.

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